BSCbooCC5030 **Seat No :________**

B.Sc. Semester - 5 (*CBCS*) Examination
Oct/Nov. - 2023(Old Course)
BOOLEAN ALGEBRA AND COMPLEX ANALYSIS-1 (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1(A) Answer any one. (07)

1. R is an equivalence relation on set X then prove that
   i. $\bigcup_{x\in X}[x] = X$
   ii. $xRy \Leftrightarrow [x] = [y]$
2. Prove that every chain is a distributive lattice.

Que-1(B) Answer any one. (04)

1. In usual notation show that (S_{30}, D) is poset.
2. Define Cover, Draw Hasse diagram of (S_{30}, D).

Que-1(C) Answer any one. (03)

1. A = {a, b, c} and $\subseteq$ is a relation of subset or equal set then show that

   P(A), $\subseteq$) is poset.
2. In usual notation prove that $a * (b * c) = (a * b) * c$

Que-2(A) Answer any one. (07)

1. If S = {a, b, c}, B = {0, 1} then show that f: P(S) → B is Boolean homomorphism for $(P(S), \cap, \cup, {}^c, \phi, S)$ and $(B, *, \oplus, ', 0, 1)$ where

   $f(x) = 1, b \in X$
   $f(x) = 0, b \notin X$
2. If $(B, *, \oplus, ', 0, 1)$ is Boolean algebra and $x_1, x_2 \in B$ then prove that

   $A(x_1 \oplus x_2) = A(x_1) \bigcup A(x_2)$.

Que-2(B) Answer any one. (04)

1. If $(B, *, \oplus, ', 0, 1)$ is finite Boolean algebra and $x \in B$, $x \neq 0$ then prove that there exist an atom a in B such that $a \leq x$.
2. For Boolean statement prove that
   $(a' * b' * c) \oplus (a * b' * c) \oplus (a * b' * c') = b' * (a \oplus c)$

Que-2(C) Answer any one. (03)

1. Define Direct product of two Boolean algebra.
2. In usual notation prove that $(a * b)' = a' \oplus b'$.

Que-3(A) Answer any one. (07)

1. In usual notation obtain Cauchy – Riemann conditions in polar form.
2. If f(z) = u + i v is an analytic function on domain D then prove that
   $\left(\frac{\partial|f(z)|}{\partial x}\right)^2 + \left(\frac{\partial|f(z)|}{\partial y}\right)^2 = |f'(z)|^2$

Que-3(B) Answer any one. (04)

1. Prove that u = log r is a harmonic function and find it's conjugate.
2. Prove that an analytic function of a constant modulus is also constant in its domain.

Que-3(C) Answer any one. (03)

1. Prove that $\frac{1}{z}$ is an analytic function but not an entire function.
2. Show that z^2 is entire function.

Que-4(A) Answer any one. (07)

1. State and prove Cauchy's integral formula.
2. Show that $\left|\int_c \frac{z+4}{z^3-1}\,dz\right| \le \frac{6\pi}{7}$ Where C be the arc of the circle $|z| = 2$ from z = 2 to z = 2i.

Que-4(B) Answer any one. (04)

1. In usual notation prove that $\left|\int_c f(z)dz\right| \le$ ML.
2. Find: $\int_c \frac{z^2+3}{z^2(z-4)}$ dz where C: $|z| = 1$.

Que-4(C) Answer any one. (03)

1. Find $\int_c \frac{z}{(z+i)}$ dz; where C: $|z| = 2$.
2. Find: $\int_c \frac{dz}{(z-3)}$ where C: $|z - 2| = 5$.

Que-5(A) Answer any one. (07)

1. State and prove the fundamental theorem of algebra.
2. State and prove Liouville's theorem.

Que-5(B) Answer any one. (04)

1. Find: $\int_c \frac{5z^2-7z+3}{(z-1)^2}$ dz where C: $|z| = 2$
2. In usual notation state and prove Cauchy's inequality.

Que-5(C) Answer any one. (03)

1. If C: $|z| = 3$ is the closed contour and the integral is fallen in positive sense and g(z) = $\int_c \frac{2s^2-s-2}{s-z}$ ds then prove that g(2) = 8πi.
2. Find: $\int_c \frac{\cosh z\, dz}{z^3}$ where C: $|z| = 1$.

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